Problems Section

A Convex Maximization Problem

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For each positive integer n, maximize the convex function

$$\sum_{i=1}^{n} \frac{1}{x_i}$$

over the polyhedron in $\mathbb{R}^n$ defined by

$$(j+1)x_j + x_i \geqslant (j+1)i + \varepsilon_{ij} \quad \text{for } 1 \leqslant j \leqslant i \leqslant n \, ,$$

where $\varepsilon_{ij} = 1$ if $i = j = 1$ and $\varepsilon_{ij} = 0$ otherwise. Prove that:

(i) a global maximum $(a_1, \ldots, a_n)$ exists and is unique
(ii) the components a_i of the global maximum satisfy $a_1 = 1$, $a_2 = 2$, $a_3 = 4$ and, when $i \geqslant 4$,

$$a_i = (j+1)(i - a_j)$$

for any j with $(j+1)a_j - ja_{j-1} \leqslant i < (j+2)a_{j+1} - (j+1)a_j$.

REMARK. A solution of this problem will imply the truth of a certain number theoretic conjecture due to Levine and O'Sullivan, *Acta Arithmetica* **34** (1977), No. 1, 9–24.

Journal of Global Optimization **2**: 419, 1992.